Financial modeling on quantum computers using digitally compressed algorithms

Financial portfolio optimization with Kipu Quantum's technology

White paper #2
Version: 1.0 – RELEASE

By Kipu Quantum GmbH, 05 December 2023

Financial modeling on quantum computers using digitally compressed algorithms – Portfolio optimization with Kipu Quantum's technology

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1. PORTFOLIO OPTIMIZATION

The sheer number of possible investments makes selecting assets for building an optimal portfolio a very difficult problem that financial institutions all over the world are tackling on a daily basis. Nevertheless, optimizing a financial portfolio in terms of maximizing value returns and minimizing the risk can be formulated as a mathematical problem that a computer can solve. Due to large amount of invested capital, solving this optimization problem with fraction of percentages improved answers, can easily results in hundreds of millions of EUR larger returns and profits. Hence, portfolio optimization is very valuable, however computationally challenging.

We consider portfolio optimization problems that belong to the class of combinatorial optimization problems[1]. Finding solutions for this problem class requires computational resources that increase exponentially with the number of considered assets. Here, the number of all possible portfolios doubles when we increase the number of considered assets by one. Due to this exponentially growing solution space, the resources required obtain the optimal solution also grow exponentially.

Classical algorithms are able to find the optimal solution up to a low couple of hundreds of assets[2], but struggle to go beyond that. However, the required computational resources can be reduced drastically by using algorithms that find an approximate solution instead of the optimal one. Using algorithms providing approximate solutions allows to optimize portfolios with 10,000 assets. Developing a method that allows tackling portfolio optimization for a larger number of assets or delivering solutions close to optimum has an important financial impact by creating larger returns and saving costs.

A very promising candidate to overcome the current limitations of portfolio optimization is using quantum computing. Quantum computing is expected to solve optimization problems much more efficiently than classical computers, enabling to tackle larger problems. Whereas a classical computer has to test possible asset combinations one by one, a quantum computer can consider many combinations at the same time, because it can use quantum effects like superposition and interference, resulting in a better performance of the quantum computer.

One possible approach to solve portfolio optimization on quantum computers is using a quantum search algorithm to find the optimal solution[3]. However, this quantum algorithm has to run on fault-tolerant quantum computers, where occurring errors are corrected. These systems use so-called logical qubits. One scalable logical qubit consists of around thousand noisy physical qubits[4] that are combined in specific ways that allow to correct for errors[5]. We can see that with the qubit overhead necessary for error-correction, optimizing a portfolio of several hundred assets would require a quantum computer with several millions of noisy qubits, which is out of reach for current hardware platforms.

In contrast, we expect that useful quantum computing for finance becomes accessible, when hundreds to thousand noisy qubits can be used in

[1] Many combinatorial optimization problems belong to the class of NP-hard problems such as the travelling salesman problem or the in this whitepaper considered form of portfolio optimization.
[2] https://doi.org/10.1155/2017/4197914
[3] Here, we refer to the famous Grover's search algorithm. https://doi.org/10.48550/arXiv.2310.03011
[4] A very recent publication demonstrated 48 logical with ~280 physical qubits. However, this very impressive approach is not suited to massively scale the logical qubits as required for textbook-like fault-tolerant quantum computing.
[5] https://doi.org/10.1073/pnas.1619152114

quantum algorithms developed by Kipu Quantum. In that regime, a quantum algorithm will be able to provide a better solution than classical algorithms or perform the optimization in a shorter time.

Table 1: **Examples of portfolio optimization with quantum computers.**

Number of assets	Algorithm type	Implementation	Approximation ratio of the solution
32	Quantum Approximate Optimization Algorithm (QAOA)	Hardware implementation on trapped ion system	0.86 (Ref. [6])
10	QAOA	Numerical simulation	0.99 (Ref. [7])
16	Variational Quantum Eigensolver	Numerical simulation	0.99 (Ref. [8])

2. IMPLEMENTATION CHALLENGES ON AVAILABLE QUANTUM HARDWARE

Unfortunately, running a quantum algorithm with hundreds of qubits on today's available quantum computers is out of reach. The reason is that the qubits as well as the computational operations, the so-called gates, building a quantum circuit are susceptible to noise inducing computational errors. In classical computers we can easily correct for errors, however correcting for errors in a quantum computer introduces a prohibitive overhead in the number of qubits.

The typically used noise-tolerant algorithms running on noisy intermediate-scale quantum (NISQ) devices are presumably limited to several tens of qubits for two reasons.

1. The first reason is that these algorithms are working with a couple of hundreds gate operations, where each gate operation introduces a tiny error that cannot be corrected. For this number of gate-operations the errors can easily become so large that one cannot extract a meaningful solution anymore.

2. The second reason is that the qubits are losing their quantum information over time due to noise induced by the environment. The time scale on which the information can be stored is called coherence time. Hence, the total time of the computation can maximally be on the same order as the coherence time to extract a solution. Since each gate-operation requires a finite time to be performed, the coherence time also applies a limit on the maximal number of gate-operations an algorithm uses. Depending on the used hardware either noise or the coherence can be limiting.

We can conclude that errors of the sources above increase with the length of the quantum circuit to which we refer as the circuit depth.

Without Kipu's technology, we estimate that solving portfolio optimization for ~200 assets with state-of-the-art quantum algorithms would require a reduction in gate errors. In addition, the qubit coherence time has to be improved significantly compared to currently available hardware. These required improvements are not totally out of reach, but are expected to occur within 3-5 years.

Nevertheless, recent examples already demonstrate the potential of using quantum computers for portfolio optimization. We list some of those examples in Table 1. Here, we especially want to highlight the results in the first row, since the corresponding algorithm was implemented on a quantum hardware.

[6] https://doi.org/10.48550/arXiv.2305.03857
[7] https://doi.org/10.1007/s11128-022-03766-5
[8] https:// doi.org/10.1109/QCS56647.2022.00017

3. HOW KIPU'S TOOLBOX IMPROVES RESULTS OF TODAY'S PROCESSORS

To heavily push the limits of what is possible on current noisy quantum computers, we at Kipu Quantum follow the spirit of developing application and hardware specific algorithms as outlined in our previous whitepapers[9,10]. The algorithms are based on our digital compression technology that allows us to reduce the circuit depth by a factor of ten to a hundred compared to state-of-the-art pure quantum algorithms. This compression is the result of Kipu's digitized-counterdiabatic quantum computing paradigm[9]. Due to the drastically reduced circuit depth, our algorithms are less susceptible to gate errors and decoherence effects of the qubits, increasing the probability to find the optimal solution. Especially when scaling up the problem size, a higher success probability directly translates into a reduced time-to-solution bringing the point when quantum computers outperform classical ones closer to reality.

Table 2: Using Kipu's technology to solve portfolio optimization on quantum computers.

Number of assets	Algorithm type	Implementation	Approximation ratio of the solution
20	Digitized Counterdiabatic Quantum Optimization (DCQO)	Hardware implementation on trapped ion system	0.85 (Ref. [11])
20	Hybrid Digitized Counterdiabatic Quantum Optimization (h-DCQO)	Hardware implementation on trapped ion system	0.87 (Ref. [11])
20	Hybrid Digitized Counterdiabatic Quantum Optimization	Numerical simulation	0.92 (Ref. [12])

4. USING KIPU'S TECHNOLOGY FOR PORTFOLIO OPTIMIZATION

In a very recent milestone, we used our technology to tackle portfolio optimization on a commercially available quantum computer. There our compression technique enabled us to implement an algorithm with 40x shorter depth than state-of-the-art purely quantum algorithms, namely digitized quantum annealing and a 2.5x shorter depth algorithm than state-of-the-art hybrid classical-quantum algorithms. For the purely quantum case, we developed an algorithm called digitized counterdiabatic quantum optimization (DCQO) which also has its hybrid classical-quantum counterpart (h-DCQO). We validate our technology by running both algorithms on the ion trap quantum computer *Aria* from *IonQ*. We list the corresponding results in Table 2.

To give some more insight into our technology, we present the workflow used to solve the portfolio optimization problem on a quantum computer in Figure 1 (a). The first steps of the workflow are the reformulation of the problem such that it can be encoded into a quantum computer. The last step is applying our digital compression technology and running the resulting quantum circuit on the hardware. In Figure 1 (b), we show schematics of quantum circuits for portfolio optimization with and without Kipu's technology. One can see that our circuits have a drastically reduced depth compared to state-of-the-art quantum algorithm.

[9] https://kipu-quantum.com/wp-content/uploads/2023/06/White-Paper-0-On-the-industrial-usefulness-of-quantum-computing.pdf
[10] https://kipu-quantum.com/wp-content/uploads/2023/06/White-Paper-1-Our-path-toward-useful-quantum-computing.pdf
[11] https://doi.org/10.1103/PhysRevResearch.4.04320
[12] https://doi.org/10.1103/PhysRevResearch.4.013141

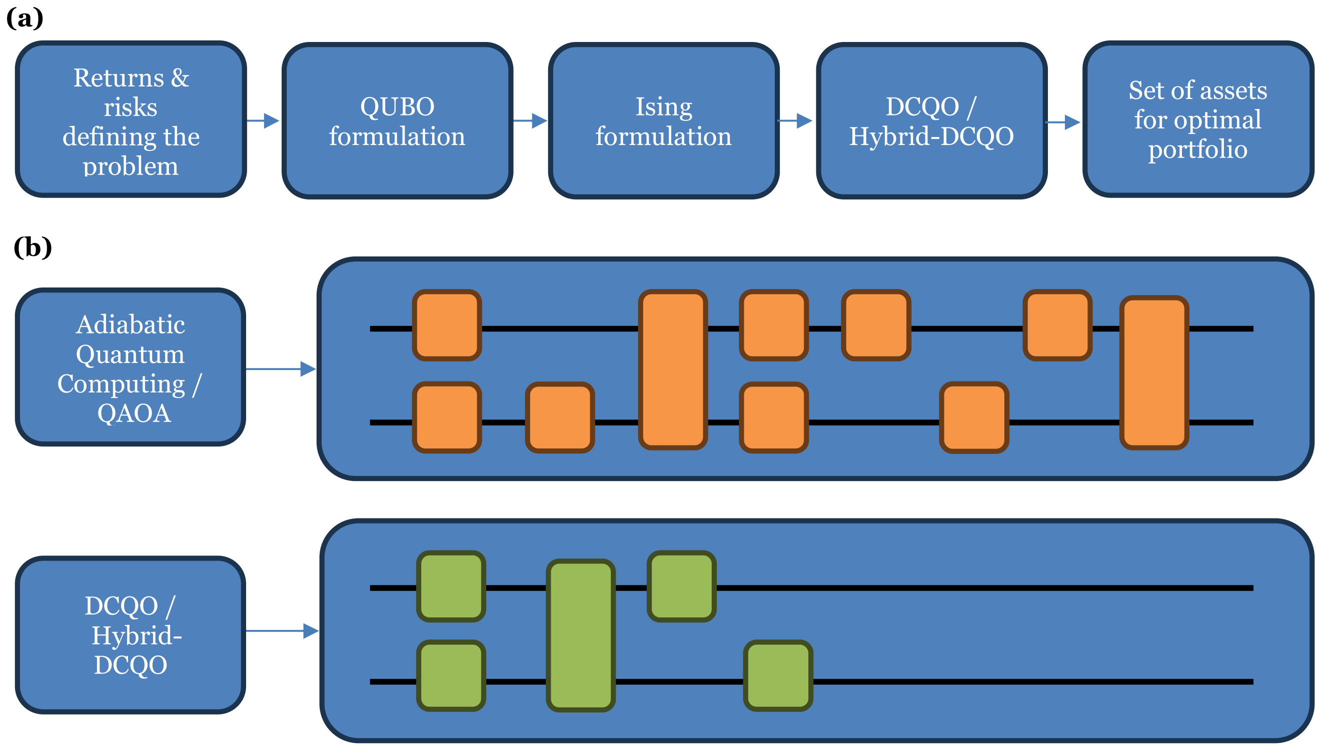

Figure 1 (a) This diagram shows the workflow to apply a quantum algorithm to solve a portfolio optimization problem at the example of our DCQO algorithms. At the first stage, we require knowledge of the expected returns of each asset and the risks coming from the covariance between assets. Secondly, the problem is defined as a quadratic unconstrained binary optimization (QUBO) problem, where the overall expected return must be maximized while minimizing the overall risk. Then, we can map the QUBO problem to an Ising formulation, which is native to a quantum computer. Finally, we can apply our algorithm to solve the problem on a quantum computer and extract the set of assets that build the optimal portfolio.

(b) Schematic of quantum computing circuits for state-of-the-art algorithms (top) versus quantum computing circuits of Kipu's algorithms (bottom). Adiabatic quantum computing and its hybrid counterpart QAOA require long circuit depths, resulting from long evolution times of the quantum system. In contrast, DCQO and its hybrid version speed up the evolution, resulting in circuit with reduced depth.

In addition, we compare the performance of our algorithms with a state-of-the-art benchmark by running both on the *Aria* quantum computer. In Figure 2, we plot the obtained approximation ratio as a function of the number of applied two-qubit gates which can be seen as a measure for the circuit depth. We see that our algorithm gives higher approximation ratio, meaning better solution quality, while requiring less two-qubit gates than the benchmark. This comparison demonstrates that our technology allows to significantly improve the quality of the solution, even on today's noisy quantum hardware.

For a more depth study we refer the interested reader to the corresponding scientific publication[9].

5. NEXT STEPS

Soon, Kipu Quantum will deliver quantum usefulness by leveraging quantum algorithms with our technology on near-team available quantum hardware. At the same time, as the hardware will improve the quality of their quantum computer, Kipu Quantum will also improve their algorithms even further. We expect to be able to demonstrate first usefulness of quantum computers for combinatorial optimization problems like portfolio optimization, when 100 qubits of realistically improved quality become available. Looking at the roadmaps of hardware providers, this stage should be reached within the next one to three years.

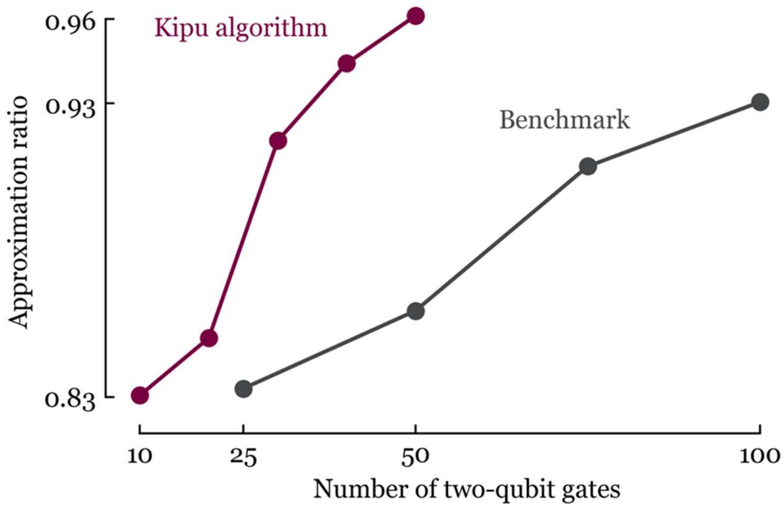

Figure 2 Comparison of Kipu's algorithm versus a state-of-the-art benchmark on hardware for 20 assets. We show the approximation ratio as a function of the number of two-gates for both algorithms. The benchmark is a state-of-the-art QAOA algorithm adapted from Ref. [5], which compare against our hybrid digitized counterdiabatic quantum optimization algorithm. Our algorithm requires less two-qubits gates while providing better solution quality. Due to the lower number of gates, our algorithm is less susceptible to hardware noise enabling us to optimize larger portfolios than the benchmark when the number of available qubits increases.

Table 3: Definition of the figures of merit used within this document

Figures of Merit	Definition	What is a good value?
Approximation ratio	The approximation ratio compares the values of the portfolio optimization problem cost function of the obtained solution with optimal solution by taking their ratio. The approximation ratio is obtained by averaging these ratios for all obtained solutions weighted by their occurrence.	1